The instrumental error in each case was not more than 1%, but the table does show that the spread in the results is over 1%, which is due to the random errors arising from visual determination.

The standard deviation may be employed [6] to determine the purely random error of measurement in each method: one assumes that the data from a series of measurements fit Student's distribution, and a confidence limit σ of 0.99 is adopted (σ is the probability that the true result falls in the confidence range), in which the results for the TI-110 indicator are as follows: first method (109 ± 1.8°C, second method 109.5 ± 1°C, and third method 109.5 ± 0.5°C.

The theory of errors [6] shows that not less than 25 measurements are required in the first method to obtain the melting point with an error of not more than 1%, as against not less than ten measurements in the second method, whereas the third method does not give a spread over 1% in any case.

This method was used to measure the switching points of indicators developed at the All-Union Luminophor Research Institute, as well as for extra-purity substances with melting points in the range 50-1200°C [7]. These indicators have been used to advantage in research on the thermal processes in the VV-12 high-throughput glass-blowing machine.

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THEORY OF A CAPILLARY VISCOMETER

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Problems in the theory of a capillary viscometer are considered where the test fluid flows through the capillary under a varying pressure drop. The effect of transients on the flow is evaluated.

Calculation of the viscosity according to the capillary method is based on the dependence of the volume V of incompressible fluid which during time τ flows through a capillary of radius R and length L on the constant pressure difference ΔP between both ends, namely

$$\eta = \frac{\pi R^4 \Delta P \tau}{8VL}. \quad (1)$$

Relation (1) was first established experimentally [1] and then confirmed theoretically on the basis of the steady-state solution to the Navier–Stokes equation [2].

In practical viscometry one often uses instruments where the test fluid flows through the capillary under a varying pressure drop $\Delta P(\tau)$. Most common among such instruments is the Golubev–Petrov viscometer [3]. With it has been measured the viscosity of water, ammonia, carbonic acid, noble gases, alcohols, and other fluids over wide ranges of states. According to the conventional method by which test data obtained with a

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TABLE 1. Characteristics of Viscometers Used for Obtaining the Experimental Data in [10]

Viscometer	$R \cdot 10^5$, m	$L \cdot 10^2$, m	$H_i \cdot 10^2$, m	During flow between	$H_f \cdot 10^2$, m	$\Delta H_{ln} \cdot 10^2$, m	$\ln \frac{H_i}{H_f} = m\tau$	$V_{mea} \cdot 10^6$, m^3
1	3,651	4,80	7,801	*I—II*	6,611	7,19	0,1654	0,3796
				I—III	5,105	6,40	0,4243	0,8480
2	3,398	5,99	7,438	*I—II*	5,534	6,48	0,2940	0,4716
				I—III	4,229	5,67	0,5632	0,7610
3	6,950	5,35	8,890	*I—II*	6,581	7,65	0,3112	0,662
				I—III	4,021	6,25	0,7949	1,338

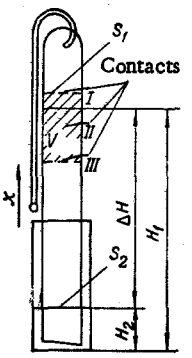

Fig. 1. Schematic diagram of the viscometer.

Golubev–Petrov viscometer are evaluated [4], calculations are made on the basis of Eq. (1) with the constant pressure drop replaced by the logarithmic mean of the initial pressure drop ΔP_i and the final pressure drop ΔP_f. This mode of averaging is based on special calibration tests involving the viscosity of water [5] and has not been justified theoretically. The exponential variation of the pressures at both ends on the capillary with time while a liquid or a gas flow through the latter under atmospheric pressure is also based on visual observations of how the meniscus of mercury [6] and that of the test fluid [7] in the viscometer drop. Meanwhile, the differences between the readings of a constant-pressure viscometer and a variable-pressure viscometer respectively do exceed the total experimental error and cast some doubt on the validity of using the steady-sate solution (1) for evaluation of data obtained under conditions of a varying pressure drop.

A theoretical expression for experimental determination of the viscosity under conditions of a varying pressure drop has been derived in earlier studies [8, 9] for a Rankin viscometer. In [8] the authors start with the Hagen–Poiseuille equation, i.e., the steady-state solution; in [9] the authors start with the differential equation of motion, but the assumptions made in the process of its solution lead to a result inconsistent with the physical nature of the process.

In this study the trend of the pressure drop as a function of time will be established by solving the differential equation of motion and subsequently analyzing the effect of transiency on that relation.

The motion of an incompressible fluid in a capillary can be described by the Navier–Stokes equation of laminar flow parallel to and symmetrically with respect to the axis

$$\frac{\eta}{\rho}\left(\frac{\partial^2 U}{\partial r^2}+\frac{1}{r}\frac{\partial U}{\partial r}\right)-\frac{1}{\rho}\frac{\Delta P(\tau)}{L}=\frac{\partial U}{\partial \tau}, \qquad (2)$$

where U is the velocity of a fluid particle at time τ at distance r from the axis of the capillary and $\Delta P(\tau)$ is the pressure drop from one end of the capillary to the other. This equation does not take into account losses of pressure on generating the kinetic energy of flow and on forming a parabolic velocity profile in the entrance zone of the capillary. These effects can be accounted for, as is done conventionally [4], if the trend of U(r) as a function of time is known. In deriving a functional relation between the pressure drop and time, we consider

a viscometer different from the conventional one in that it has a cylindrical test space and intermediate contacts (Fig. 1). Viscometers of such a construction (their geometrical parameters are given in Table 1) have been used for measuring the viscosity of several Freons and of nitrogen at various temperatures and pressures [10]. The errors in recording the mercury level in the wide arm of the test space can be greatly reduced by automating the process of viscometer calibration. Stability of separation of the mercury meniscus from the contacts was quite satisfactorily confirmed by the reproducibility (within 0.2-0.3%) of the experimental values.

The volume dV of fluid flowing in the capillary and the change of pressure drop $d\Delta P(\tau)$ between the ends of the capillary occurring in time $d\tau$ can be expressed in terms of changes in the mercury levels H_1 and H_2, namely

$$dV = -d(H_1 - H_2)\frac{1}{\frac{1}{S_1}+\frac{1}{S_2}}, \tag{3}$$

$$d\Delta P(\tau) = d(H_1 - H_2)\, g\rho_{Hg}, \tag{4}$$

where S_1 and S_2 are the cross-sectional areas of the two cylindrical containers of the viscometer. Meanwhile,

$$dV = \int_0^R 2\pi U r dr. \tag{5}$$

Expressions (3), (4), and (5) inserted into Eq. (2) transform the latter to

$$\frac{\eta}{\rho}\left(\frac{\partial^2 U}{\partial r^2}+\frac{1}{r}\frac{\partial U}{\partial r}\right)-\frac{2\pi g\rho_{Hg}}{\rho L}\left(\frac{1}{S_1}+\frac{1}{S_2}\right)\int_0^\tau\int_0^R U r dr d\tau = \frac{\partial U}{\partial \tau}. \tag{6}$$

An analogous equation was obtained in [9] for describing the flow in a Rankin viscometer. Owing to the mathematical difficulties in obtaining an exact solution, an approximate solution was obtained in [9] indicating an oscillatory variation of the pressure drop during gravity flow through the capillary – inconsistent with the physical nature of the phenomenon. This result can be explained as a consequence of seeking in [9] a solution to an equation like (6) in the form of a double infinite series in powers of the parameter $\lambda = \eta/\rho R^2$, although this is not a small parameter in viscosity measurements.

In order to obtain an approximate solution to the integrodifferential equation (6), it will be suggested here that the acceleration $\partial U/\partial\tau$ due to a change of pressure $\Delta P(\tau)$ drop during flow is negligible in comparison with the sectional velocity gradient due to viscous forces. Then

$$\frac{\eta}{\rho}\left(\frac{\partial^2 U_1}{\partial r^2}+\frac{1}{r}\frac{\partial U_1}{\partial r}\right)-\frac{2\pi g\rho_{Hg}}{\rho L}\left(\frac{1}{S_1}+\frac{1}{S_2}\right)\int_0^\tau\int_0^R U_1 r dr d\tau = 0, \tag{7}$$

where U_1 is the approximate value of velocity U on the assumption that $\partial U/\partial\tau = 0$. If at the instant $\tau = 0$, some time after the fluid has physically begun to move in the capillary, the pressure drop from one end to the other end of this capillary is ΔP_i, then

$$U_1(r, 0) = \frac{\Delta P_i}{4\eta L}(R^2 - r^2). \tag{7'}$$

The boundary conditions here are determined by adhesion of the fluid to the capillary wall and by the finiteness of the fluid velocity along the capillary axis, namely

$$U_1(R, \tau) = 0, \quad U_1(0, \tau) \neq \infty. \tag{7''}$$

The solution to the quasisteady problem formulated according to Eq. (7) with the initial condition (7') and the boundary conditions (7") is

$$U_1 = \frac{2R^2}{\eta L}\Delta P_i \exp(-m\tau)\sum_1^\infty \frac{J_0(\rho_f r)}{\mu_f^3 J_1(\mu_f)} \quad \text{or} \quad U_1 = \tag{8}$$

$$= \frac{\Delta P_i}{4\eta L}\exp(-m\tau)(R^2 - r^2),$$

TABLE 2. Some Results of Experimental Determination Pertaining to the Viscosity of Liquids and Gases, and of Calculations Pertaining to the Characteristic Flow Numbers

Substance	Parameters T/T_{cr}	Parameters P/P_{cr}	$\eta \cdot 10^7$, Pa·sec	ρ, kg/m^3	τ, sec	ν/R^2, 1/sec	m, 1/sec	$\frac{m}{\nu/R^2} \cdot 100\%$	N_{Re}	$N_{Ho} \cdot 10^{-6}$
F12V1	0,57	2,2	6200	2010	272	62,8	0,0011	0,002	78,6	0,67
F22	0,81	0,22	1620	1190	71,4	26,9	0,004	0,01	670	0,67
F13V1	0,86	0,38	1500	1570	69,8	19,5	0,004	0,02	987	0,67
F23	0,85	1,04	1313	1193	58,8	22,4	0,005	0,02	1017	0,67
Nitrogen	3,2	0,03	229	0,87	64,0	19500	0,003	0,001	3,9	2,49
Nitrogen	3,2	0,8	233	22,4	66,3	780	0,003	0,001	90	2,49
F502	1,02	1,21	407	625	124	48,4	0,001	0,003	760	2,49
F502	0,94	0,5	157	102	46	116	0,004	0,003	870	2,49
Nitrogen	2,3	0,03	178	1,25	111,6	12300	0,003	0,001	5,4	3,7
F12V1	0,64	0,02	120	7,0	76,3	1520	0,004	0,001	70	3,7
F13V1	1,05	2,5	930	1300	663	61,8	0,0005	0,001	181	3,7
F13V1	1,003	0,99	260	515	179	43,6	0,002	0,004	948	3,7

*F indicates Freon.

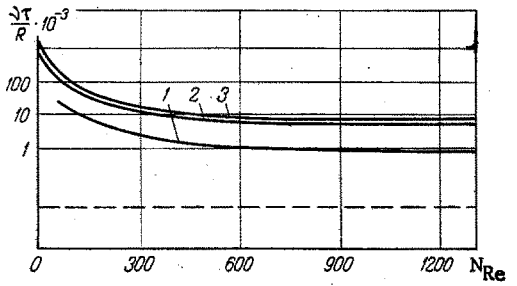

Fig. 2. Dependence of the number $\nu\tau/R^2$ on the number N_{Re}, at various values of the homochronism number N_{Ho}: 1) $0.67 \cdot 10^6$; 2) $2.49 \cdot 10^6$; 3) $3.70 \cdot 10^6$.

where

$$m = \frac{4\pi g \rho_{Hg} R^4}{\eta L}\left(\frac{1}{S_1} + \frac{1}{S_2}\right)\sum_1^\infty \frac{1}{\mu_f^4};$$

and $p_f = \mu_f/R$ are roots of the Bessel function $J_0(r/R)$. This together with Eqs. (2), (3), and (5) yields

$$\Delta P(\tau) = \Delta P_i \exp(-m\tau). \tag{9}$$

Consequently, a logarithmic averaging of the pressure drop over time in the case of a quasisteady flow is correct only for viscometers with a cylindrical test space and this has been confirmed experimentally [6, 7].

In order to estimate the error due to disregarding the transiency when Eq. (6) is replaced with Eq. (7), one has to insert into Eq. (2) the value of $\Delta P(\tau)$ according to expression (9)

$$\nu\left(\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r}\right) + \frac{\Delta P_i}{\rho L}\exp(-m\tau) = \frac{\partial U}{\partial \tau}. \tag{10}$$

The initial condition is (7') and the boundary conditions are (7"). The solution to the second-order linear differential equation with the transient inhomogeneity (10) is

$$U(r, \tau) = \frac{2R^2\Delta P_i}{\eta L}\sum_1^\infty \frac{J_0(p_f r)}{\mu_f^3 J_1(\mu_f)}\exp(-\nu p_f^2\tau) + \sum_1^\infty \frac{2J_0(p_f r)}{\mu_f J_1(\mu_f)} \times$$
$$\times \frac{\Delta P_i}{\rho L}\frac{1}{\nu p_f^2 - m}[\exp(-m\tau) - \exp(-\nu p_f^2\tau)]. \tag{11}$$

The first term of expression (11) describes the radial velocity profile in the capillary at the time zero and decreases proportionally to $\exp(-\nu p_f^2 \tau)$:

$$\nu p_f^2 \tau = \frac{\nu\tau}{R^2} \mu_f^2 = \frac{2N_{Ho}}{N_{Re}} \mu_f^2. \tag{12}$$

An evaluation of (ν/R^2) from viscosity data for several Freons [10] and for nitrogen (Table 2) reveals that $\exp(-\nu p_f^2 \tau)$ tends to zero for $\tau \geq 0.05$ sec. In the physical sense the first term characterizes the change in velocity of laminar flow following a step change of the driving force from some magnitude to zero. A comparative evaluation of quantities ν/R^2 and m (Table 2) in the second term of expression (11) suggests that, within an accuracy better than 0.02%, m can be regarded relative to ν/R^2. Consequently, the quasisteady solution (8) and the transient solution (11) to the integrodifferential equation (6) become equivalent when the characteristic flow numbers $\nu\tau/R^2$ and N_{Ho} are within the range of values indicated in Fig. 2. Here the dashed lines represent the limiting values of $\nu\tau/R^2$ below which disregarding the transiency will result in an error larger than 0.03%.

It appears from the preceding discussion that flow characterized by certain values of numbers $\nu\tau/R^2$ and N_{Ho} can be described by the quasisteady model without, for all practical purposes, affecting the accuracy of experimental results. An exponential variation of the pressure drop with time during quasisteady flow is correct to assume also for viscometers of other constructions, as long as the test space in them has a uniform cross section throughout its height, as has been confirmed experimentally [6, 7].

In practical calculations of the viscosity from experimental data it is necessary to add corrections for the kinetic energy as well as for the turbulence of flow at the entrance to the capillary

$$\Delta P_1 = m_1 \rho \bar{U}^2, \quad \bar{U} = \frac{1}{\tau} \int_0^\tau \frac{1}{2} U(0, \tau)\, d\tau, \tag{13}$$

where the empirical coefficient m_1 accounts for loss of pressure within the entrance zone of the capillary and $\bar{U}$ represents the average mean, over time and cross section, velocity of the fluid. Together with solution (8), then, relations (13) yield the conventional [4] correction for the kinetic energy and the existence of an entrance zone.

NOTATION

ν, η, ρ	are respectively the kinematic viscosity, the dynamic viscosity, and the density of the fluid flowing through the capillary;
ρ_{Hg}	is the density of mercurcy;
g	is the acceleration of free fall;
P	is the test pressure;
T	is the test temperature;
P_{cr} and T_{cr}	are the pressure and the temperature at the critical point;
H_1 and H_2	are the mercury levels in the viscometer at time τ;
H_i and H_f	are the initial and the final mercury levels in the viscometer;
ΔH_{ln}	is the logarithmic mean difference of mercury levels during flow;
V_{mea}	is the volume of the test space;
R	is the capillary radius;
L	is the capillary length;
ΔP	is the pressure drop from one end of the capillary to the other;
μ_φ	is the root of the Bessel function $J_0(r)$;
N_{Re}	is the Reynolds number;
N_{Ho}	is the homochronism number.

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CHARACTERISTICS OF A FLOW OF MONODISPERSE GAS–LIQUID MIXTURE IN A VERTICAL TUBE

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UDC 532.529.5

The results of measurement of the wall shear stress, and the void and liquid-velocity profiles in an ascending two-phase flow containing gas bubbles of uniform size are given. It is shown that the bubble size has a significant effect on the flow structure and characteristics.

The need to investigate the fine structure of two-phase flows is due to the complex motion of the phases and to the large number of parameters that affect the characteristics of such flows. Several experimental studies [1-3] have shown that there may be different local void distributions over the tube cross section, which must obviously affect other characteristics of the flow (velocity profile, wall shear stress, heat- and mass-transfer coefficients). Bubbly flow, which is often encountered in engineering applications, has been most poorly investigated so far. The presently available and very few measurements of the friction factor in these conditions [4-6] indicate the presence of a region of sharp increase in the wall shear stress in comparison with single-phase flow at low values of void fraction. The relations $\tau/\tau_0(\beta)$ were not identical in the different investigations and in a number of cases a single relation could not be obtained even in the same experiment [5, 6]. It follows from this that in a particular region of parameters we do not have sufficient knowledge of the

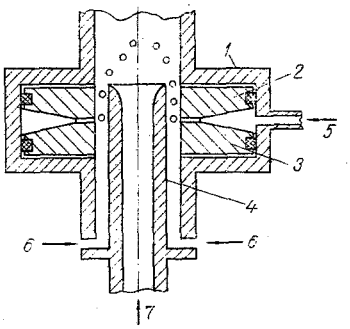

Fig. 1. Generator of calibrated gas bubbles: 1) case; 2, 3) rings; 4) insert; 5) gas input; 6) slit liquid input; 7) central liquid input.

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